Passage 24 A #4

- ## Population growth when annual growth rate is not a constant

Let r be annual growth rate.

$$N_{t+1} = r N_t$$

If r is a constant and $r > 1$, the population expondntiall increases. If $0 < r < 1$, it exponentially decreases to zero.

But annual growth rate r may not be a constant. It likely changes with time; economy, social system,etc. would influence it. How does the population change when annual growth rate changes with time?

In this course, we numerically simulate population dynamics for 1) Exponential growth (annual growth rate being constant), 2) Annual growth rate linearly decreases with time, 3) It linearly increases with time.

## Annaual growth rate is a constant - exponential model

```
tPeriod = 50
```

50

The population increases by a constant percentage every year.

```
seqGrowthRate0 = Table[1.05, {t, tPeriod}]
```

{1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05,
 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05,
 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05,
 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05}

```
ListPlot[seqGrowthRate0, AxesLabel → {"t", "r"}]
```
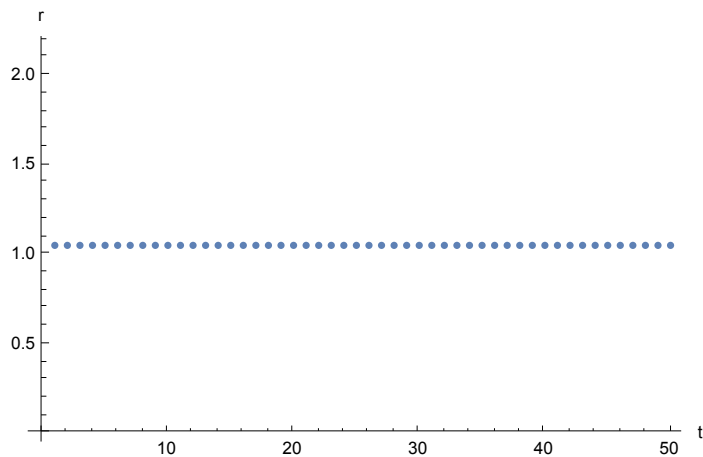


```
data0 = {};
popSize = 1;

Do[
 AppendTo[data0, popSize];
 popSize = popSize * seqGrowthRate0[[i]];, {i, 1, tPeriod}
]
```

```
data0
```

{1, 1.05, 1.1025, 1.15763, 1.21551, 1.27628, 1.3401, 1.4071, 1.47746, 1.55133,
 1.62889, 1.71034, 1.79586, 1.88565, 1.97993, 2.07893, 2.18287, 2.29202,
 2.40662, 2.52695, 2.6533, 2.78596, 2.92526, 3.07152, 3.2251, 3.38635,
 3.55567, 3.73346, 3.92013, 4.11614, 4.32194, 4.53804, 4.76494, 5.00319,
 5.25335, 5.51602, 5.79182, 6.08141, 6.38548, 6.70475, 7.03999, 7.39199,
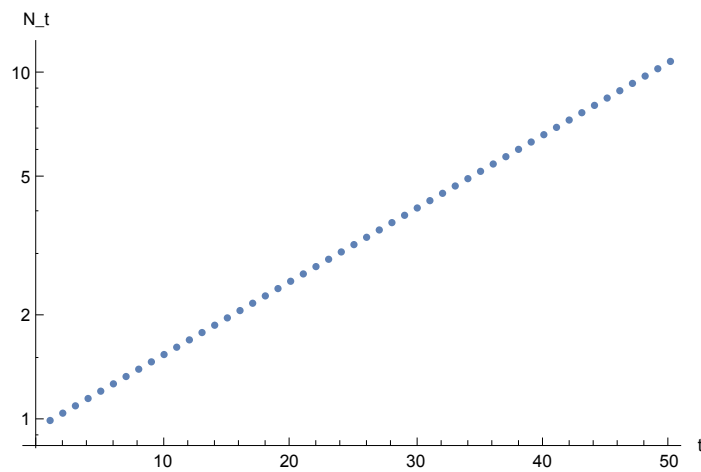 7.76159, 8.14967, 8.55715, 8.98501, 9.43426, 9.90597, 10.4013, 10.9213}

```
Log[2] / Log[1.05]
```

14.2067

```
g0 = ListPlot[data0, AxesLabel → {"t", "N_t"}]
```
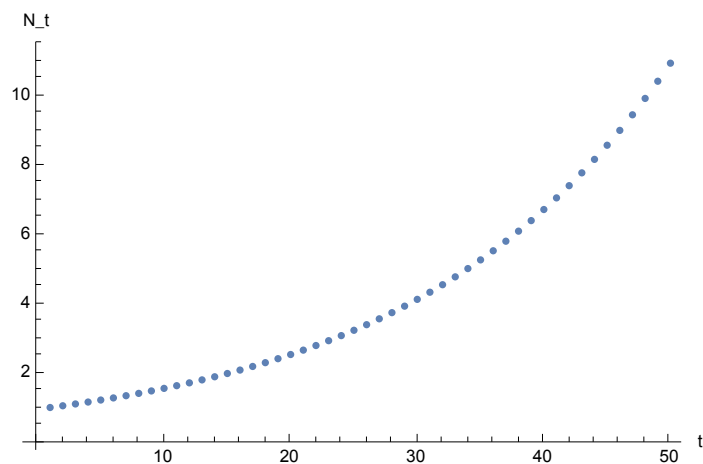
```
gLog0 = ListLogPlot[data0, AxesLabel → {"t", "N_t"}]
```



Annaual growth rate linearly decreases with time

```
tPeriod = 50
```

50

```
seqGrowthRate1 = Table[1.05 - 0.001 (t - 1), {t, tPeriod}]
```

{1.05, 1.049, 1.048, 1.047, 1.046, 1.045, 1.044, 1.043, 1.042, 1.041,
 1.04, 1.039, 1.038, 1.037, 1.036, 1.035, 1.034, 1.033, 1.032, 1.031,
 1.03, 1.029, 1.028, 1.027, 1.026, 1.025, 1.024, 1.023, 1.022, 1.021,
 1.02, 1.019, 1.018, 1.017, 1.016, 1.015, 1.014, 1.013, 1.012, 1.011,
 1.01, 1.009, 1.008, 1.007, 1.006, 1.005, 1.004, 1.003, 1.002, 1.001}

```
ListPlot[seqGrowthRate1, AxesLabel → {"t", "r"}]
```
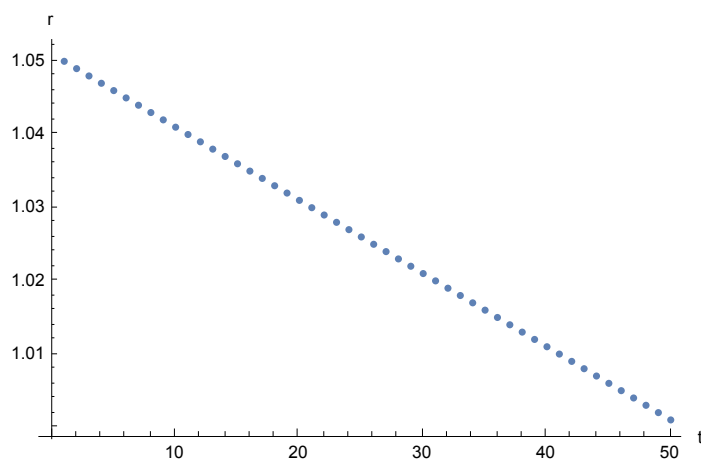


```
data1 = {};
popSize = 1;

Do[
 AppendTo[data1, popSize];
 popSize = popSize * seqGrowthRate1[[i]];, {i, 1, tPeriod}
]
```
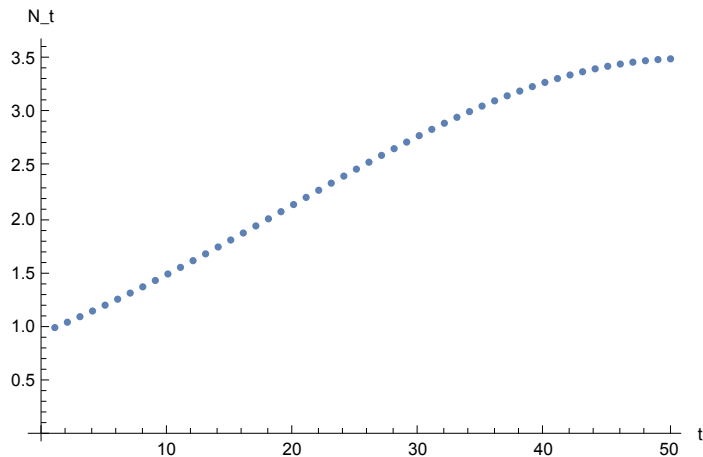
data1

{1, 1.05, 1.10145, 1.15432, 1.20857, 1.26417, 1.32105, 1.37918, 1.43849, 1.4989,
 1.56036, 1.62277, 1.68606, 1.75013, 1.81488, 1.88022, 1.94603, 2.01219,
 2.0786, 2.14511, 2.21161, 2.27796, 2.34402, 2.40965, 2.47471, 2.53905,
 2.60253, 2.66499, 2.72629, 2.78626, 2.84477, 2.90167, 2.9568, 3.01002,
 3.0612, 3.11017, 3.15683, 3.20102, 3.24264, 3.28155, 3.31764, 3.35082,
 3.38098, 3.40803, 3.43188, 3.45247, 3.46974, 3.48361, 3.49407, 3.50105}

g1 = ListPlot[data1, AxesLabel → {"t", "N_t"}]

gLog1 = ListLogPlot[data1, AxesLabel → {"t", "N_t"}]
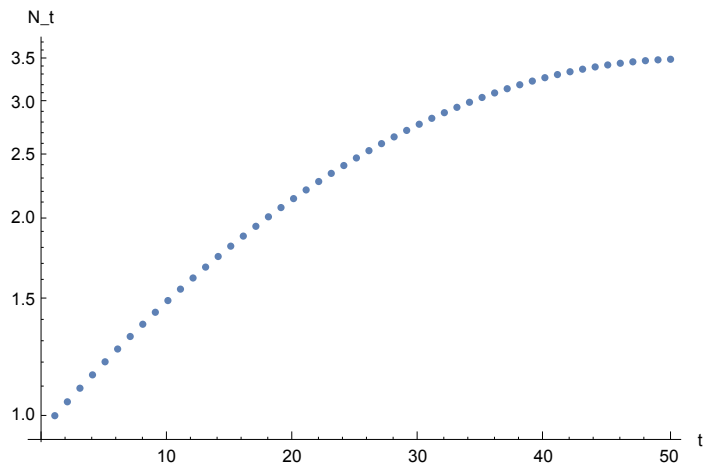

Annaual growth rate linearly increases with time

tPeriod = 50

50

seqGrowthRate2 = Table[1.05 + 0.001 (t – 1), {t, tPeriod}]

{1.05, 1.051, 1.052, 1.053, 1.054, 1.055, 1.056, 1.057, 1.058, 1.059,
 1.06, 1.061, 1.062, 1.063, 1.064, 1.065, 1.066, 1.067, 1.068, 1.069,
 1.07, 1.071, 1.072, 1.073, 1.074, 1.075, 1.076, 1.077, 1.078, 1.079,
 1.08, 1.081, 1.082, 1.083, 1.084, 1.085, 1.086, 1.087, 1.088, 1.089,
 1.09, 1.091, 1.092, 1.093, 1.094, 1.095, 1.096, 1.097, 1.098, 1.099}

```
ListPlot[seqGrowthRate2, AxesLabel → {"t", "r"}]
```
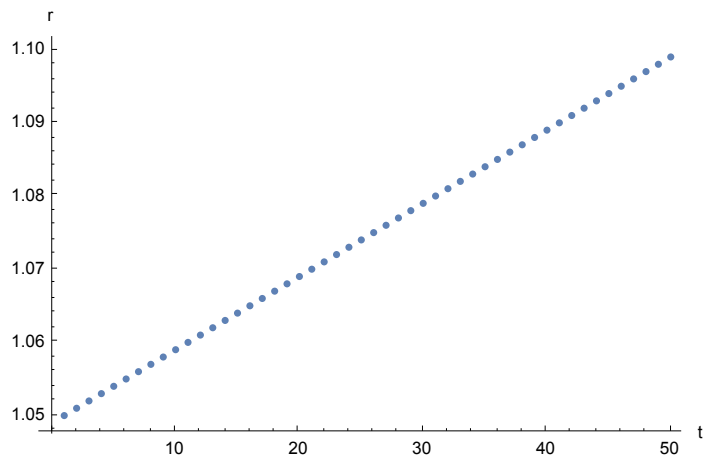


```
data2 = {};
popSize = 1;

Do[
 AppendTo[data2, popSize];
 popSize = popSize * seqGrowthRate2[[i]];, {i, 1, tPeriod}
]
```

```
data2
```

{1, 1.05, 1.10355, 1.16093, 1.22246, 1.28848, 1.35934, 1.43547, 1.51729,
 1.60529, 1.7, 1.802, 1.91193, 2.03046, 2.15838, 2.29652, 2.44579, 2.60722,
 2.7819, 2.97107, 3.17607, 3.3984, 3.63969, 3.90174, 4.18657, 4.49638,
 4.8336, 5.20096, 5.60143, 6.03834, 6.51537, 7.0366, 7.60657, 8.23031,
 8.91342, 9.66215, 10.4834, 11.385, 12.3755, 13.4645, 14.6629, 15.9826,
 17.437, 19.0412, 20.812, 22.7683, 24.9313, 27.3247, 29.9752, 32.9128}
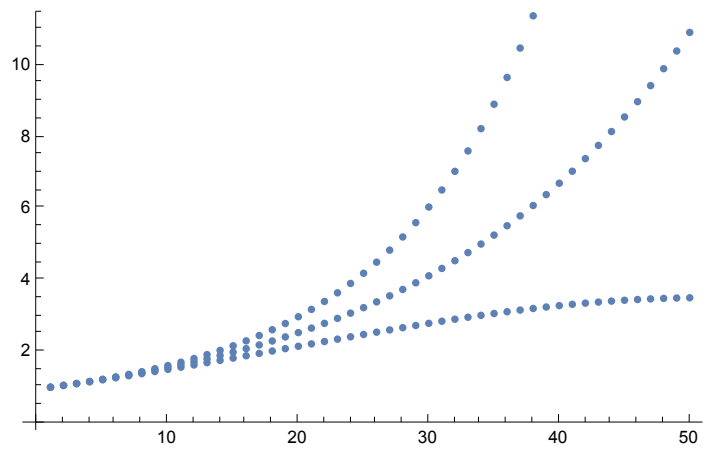
```
g2 = ListPlot[data2, AxesLabel → {"t", "N_t"}]
```
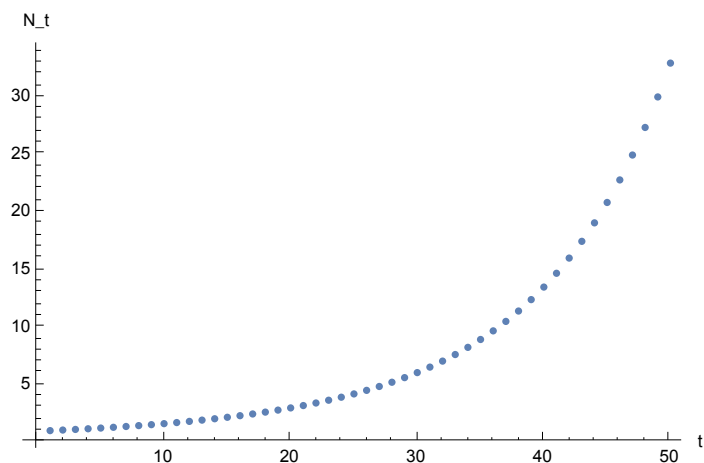
```
gLog2 = ListLogPlot[data2, AxesLabel → {"t", "N_t"}]
```
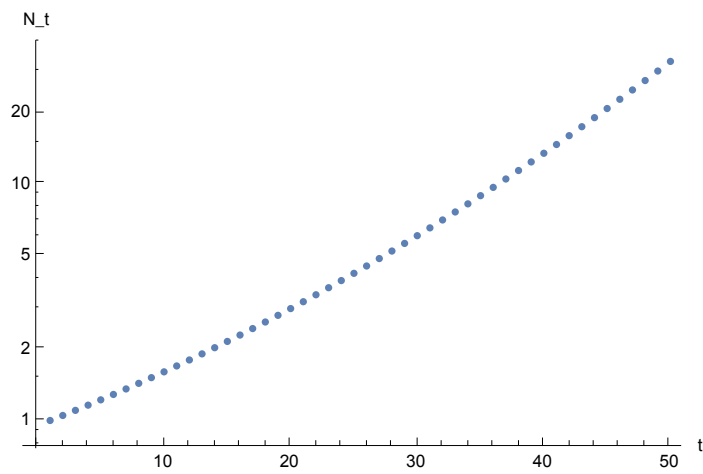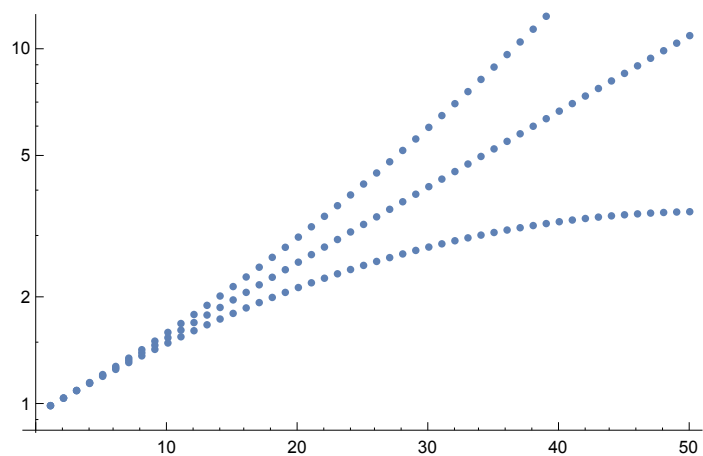

Let compare the three cases

```
Show[g0, g1, g2]
```



```
Show[gLog0, gLog1, gLog2]
```


Question

Assume various scenarios about annual growth rate that changes with time and simulate it.

For example, annual growth rate of the world population was 1.07% in 2015. If it linearly decreases by e.g., 0.02% every year, how does the world pouplation grow in future? What is the maximal world population in this case?

Annual growth rate of Japanese population was - 0.17 % in 2015. If it linearly increases by e.g., 0.02 % every year, when does Japanese population start to increase? What is the minimal Japanese population in this case?

What else would be interesting? Simulate them.

Obsolete