# Passage 24 A #4

## Population growth when annual growth rate is not a constant

Let $r$ be annual growth rate.

$$N_{t+1} = r\,N_t$$

If $r$ is a constant and $r > 1$, the population expondntiall increases. If $0 < r < 1$, it exponentially decreases to zero.

But annual growth rate $r$ may not be a constant. It likely changes with time; economy, social system,etc. would influence it. How does the population change when annual growth rate changes with time?

In this course, we numerically simulate population dynamics for 1) Exponential growth (annual growth rate being constant), 2) Annual growth rate linearly decreases with time, 3) It linearly increases with time.

---

## Annual growth rate is a constant - exponential model

```
tPeriod = 50
```

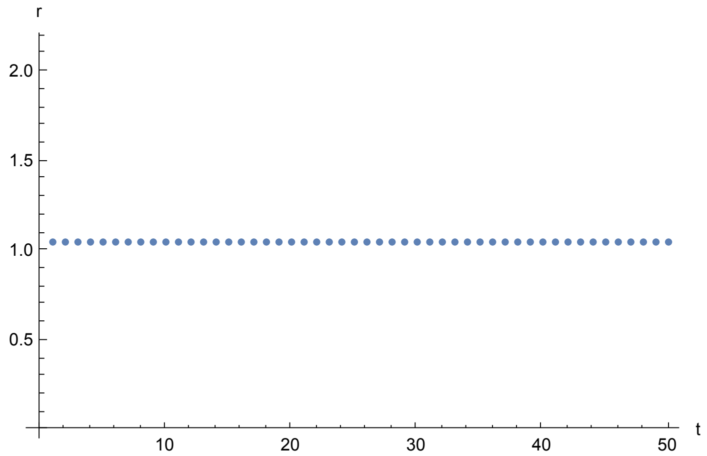
```
50
```

The population increases by a constant percentage every year.

```
seqGrowthRate0 = Table[1.05, {t, tPeriod}]
```

```
{1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05,
 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05,
 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05,
 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05}
```

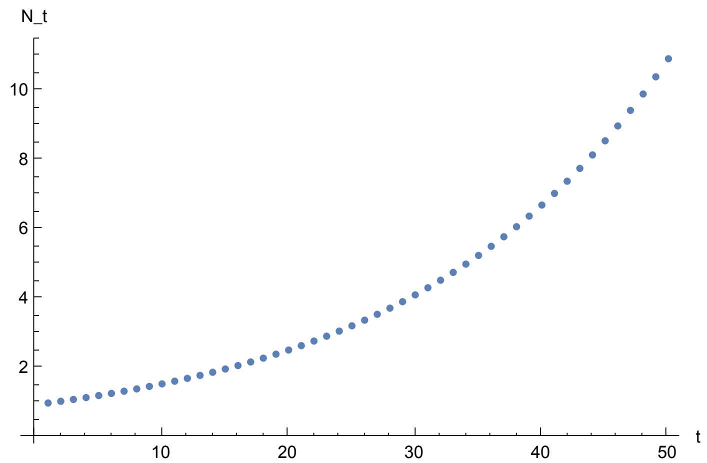
```
ListPlot[seqGrowthRate0, AxesLabel → {"t", "r"}]
```



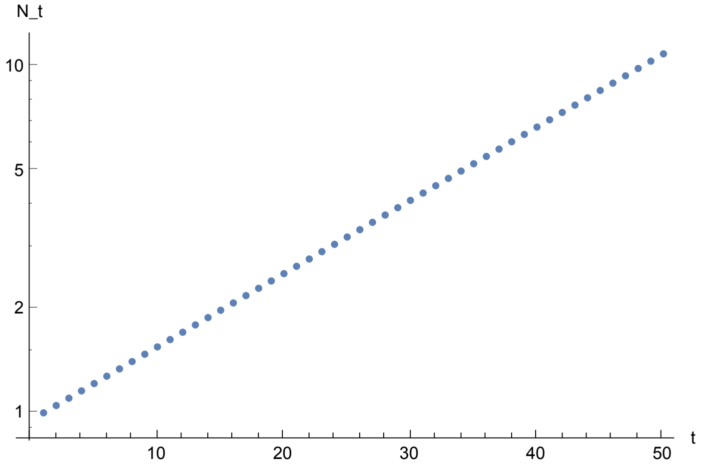

```
data0 = {};
popSize = 1;

Do[
 AppendTo[data0, popSize];
 popSize = popSize * seqGrowthRate0[[i]];, {i, 1, tPeriod}
]

data0
```

```
{1, 1.05, 1.1025, 1.15763, 1.21551, 1.27628, 1.3401, 1.4071, 1.47746, 1.55133,
 1.62889, 1.71034, 1.79586, 1.88565, 1.97993, 2.07893, 2.18287, 2.29202,
 2.40662, 2.52695, 2.6533, 2.78596, 2.92526, 3.07152, 3.2251, 3.38635,
 3.55567, 3.73346, 3.92013, 4.11614, 4.32194, 4.53804, 4.76494, 5.00319,
 5.25335, 5.51602, 5.79182, 6.08141, 6.38548, 6.70475, 7.03999, 7.39199,
 7.76159, 8.14967, 8.55715, 8.98501, 9.43426, 9.90597, 10.4013, 10.9213}
```

```
g0 = ListPlot[data0, AxesLabel → {"t", "N_t"}]
```



```
gLog0 = ListLogPlot[data0, AxesLabel → {"t", "N_t"}]
```




```
Log[2] / Log[1.05]
```

14.2067

# Annual growth rate linearly decreases with time

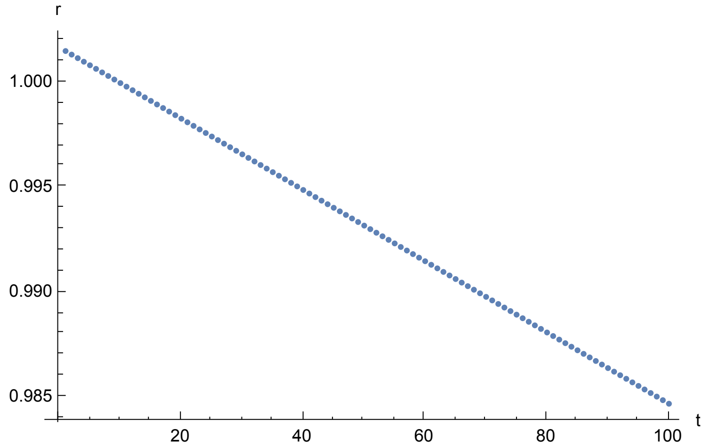
```
tPeriod = 100
```

100

```
seqGrowthRate1 = Table[1 + 0.0015 - 0.00017 (t - 1), {t, tPeriod}]
```

{1.0015, 1.00133, 1.00116, 1.00099, 1.00082, 1.00065, 1.00048, 1.00031, 1.00014,
0.99997, 0.9998, 0.99963, 0.99946, 0.99929, 0.99912, 0.99895, 0.99878, 0.99861,
0.99844, 0.99827, 0.9981, 0.99793, 0.99776, 0.99759, 0.99742, 0.99725, 0.99708,
0.99691, 0.99674, 0.99657, 0.9964, 0.99623, 0.99606, 0.99589, 0.99572,
0.99555, 0.99538, 0.99521, 0.99504, 0.99487, 0.9947, 0.99453, 0.99436,
0.99419, 0.99402, 0.99385, 0.99368, 0.99351, 0.99334, 0.99317, 0.993, 0.99283,
0.99266, 0.99249, 0.99232, 0.99215, 0.99198, 0.99181, 0.99164, 0.99147,
0.9913, 0.99113, 0.99096, 0.99079, 0.99062, 0.99045, 0.99028, 0.99011,
0.98994, 0.98977, 0.9896, 0.98943, 0.98926, 0.98909, 0.98892, 0.98875,
0.98858, 0.98841, 0.98824, 0.98807, 0.9879, 0.98773, 0.98756, 0.98739,
0.98722, 0.98705, 0.98688, 0.98671, 0.98654, 0.98637, 0.9862, 0.98603,
0.98586, 0.98569, 0.98552, 0.98535, 0.98518, 0.98501, 0.98484, 0.98467}

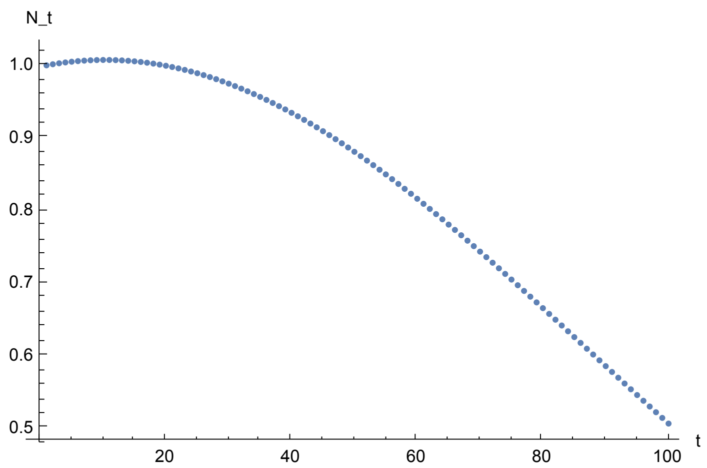
```
ListPlot[seqGrowthRate1, AxesLabel → {"t", "r"}]
```



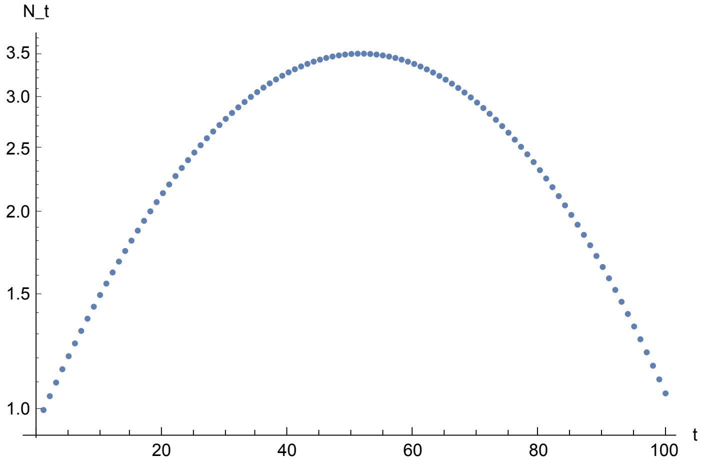

```
data1 = {};
popSize = 1;

Do[
 AppendTo[data1, popSize];
 popSize = popSize * seqGrowthRate1[[i]];, {i, 1, tPeriod}
]

data1
```

```
{1, 1.0015, 1.00283, 1.004, 1.00499, 1.00581, 1.00647, 1.00695, 1.00726, 1.0074,
 1.00737, 1.00717, 1.0068, 1.00626, 1.00554, 1.00466, 1.0036, 1.00238,
 1.00098, 0.999422, 0.997693, 0.995797, 0.993736, 0.99151, 0.98912, 0.986569,
 0.983856, 0.980983, 0.977951, 0.974763, 0.97142, 0.967923, 0.964274,
 0.960474, 0.956527, 0.952433, 0.948195, 0.943814, 0.939293, 0.934634,
 0.92984, 0.924911, 0.919852, 0.914664, 0.90935, 0.903912, 0.898353, 0.892675,
 0.886882, 0.880975, 0.874958, 0.868834, 0.862604, 0.856272, 0.849842,
 0.843315, 0.836695, 0.829985, 0.823187, 0.816305, 0.809342, 0.802301,
 0.795185, 0.787996, 0.780739, 0.773415, 0.766029, 0.758583, 0.751081,
 0.743525, 0.735919, 0.728265, 0.720568, 0.712829, 0.705052, 0.69724, 0.689396,
 0.681523, 0.673624, 0.665702, 0.65776, 0.649801, 0.641828, 0.633844, 0.625851,
 0.617853, 0.609852, 0.60185, 0.593852, 0.585859, 0.577873, 0.569899, 0.561937,
 0.553991, 0.546064, 0.538157, 0.530273, 0.522414, 0.514583, 0.506782}
```

```
g1 = ListPlot[data1, AxesLabel → {"t", "N_t"}]
```



```
gLog1 = ListLogPlot[data1, AxesLabel → {"t", "N_t"}]
```




# Annual growth rate linearly increases with time

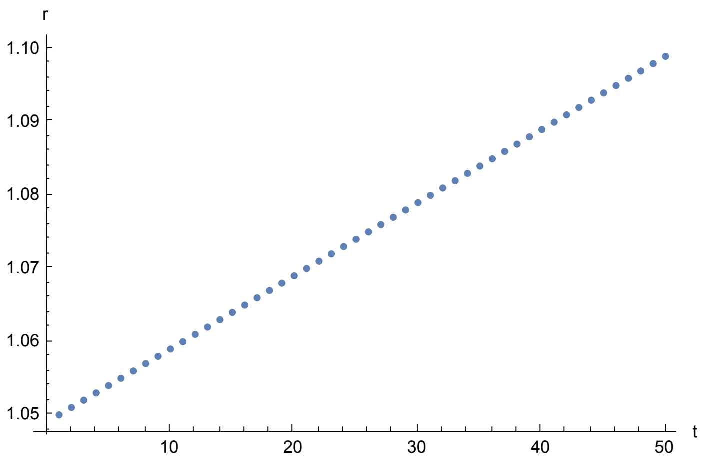
```
tPeriod = 50
```

```
50
```

```
seqGrowthRate2 = Table[-0.08 + 0.001 (t - 1), {t, tPeriod}]
```

```
{1.05, 1.051, 1.052, 1.053, 1.054, 1.055, 1.056, 1.057, 1.058, 1.059,
 1.06, 1.061, 1.062, 1.063, 1.064, 1.065, 1.066, 1.067, 1.068, 1.069,
 1.07, 1.071, 1.072, 1.073, 1.074, 1.075, 1.076, 1.077, 1.078, 1.079,
 1.08, 1.081, 1.082, 1.083, 1.084, 1.085, 1.086, 1.087, 1.088, 1.089,
 1.09, 1.091, 1.092, 1.093, 1.094, 1.095, 1.096, 1.097, 1.098, 1.099}
```

```
ListPlot[seqGrowthRate2, AxesLabel → {"t", "r"}]
```

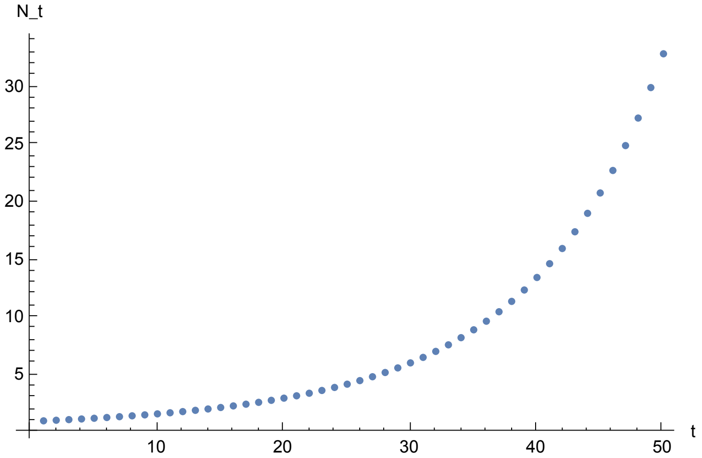



```
data2 = {};
popSize = 1;

Do[
 AppendTo[data2, popSize];
 popSize = popSize * seqGrowthRate2[[i]];, {i, 1, tPeriod}
]
```

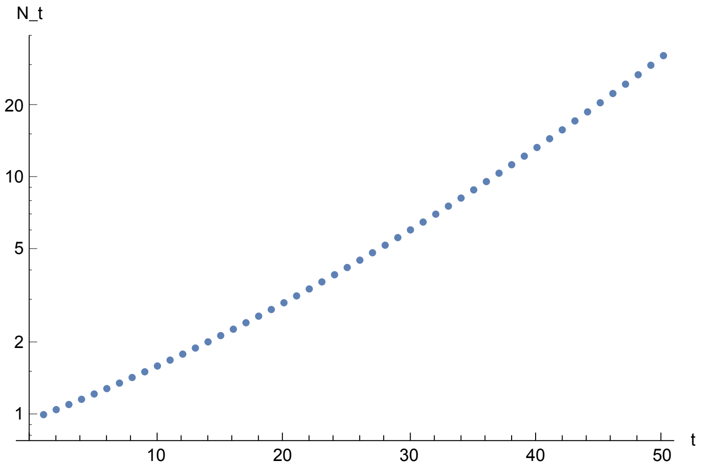
```
data2
```

```
{1, 1.05, 1.10355, 1.16093, 1.22246, 1.28848, 1.35934, 1.43547, 1.51729,
 1.60529, 1.7, 1.802, 1.91193, 2.03046, 2.15838, 2.29652, 2.44579, 2.60722,
 2.7819, 2.97107, 3.17607, 3.3984, 3.63969, 3.90174, 4.18657, 4.49638,
 4.8336, 5.20096, 5.60143, 6.03834, 6.51537, 7.0366, 7.60657, 8.23031,
 8.91342, 9.66215, 10.4834, 11.385, 12.3755, 13.4645, 14.6629, 15.9826,
 17.437, 19.0412, 20.812, 22.7683, 24.9313, 27.3247, 29.9752, 32.9128}
```

```
g2 = ListPlot[data2, AxesLabel → {"t", "N_t"}]
```

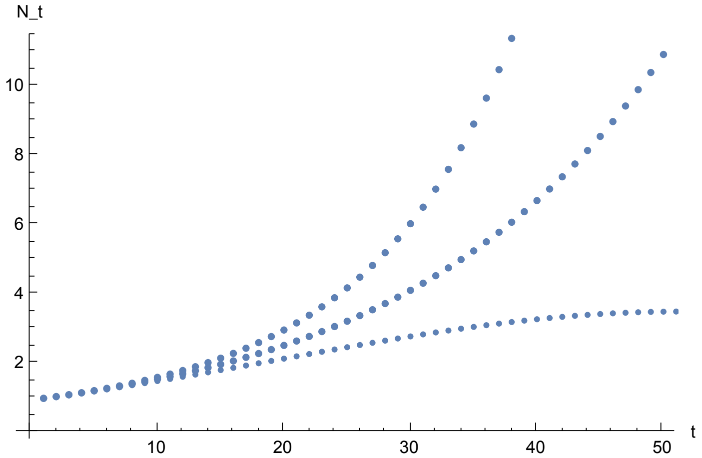



```
gLog2 = ListLogPlot[data2, AxesLabel → {"t", "N_t"}]
```

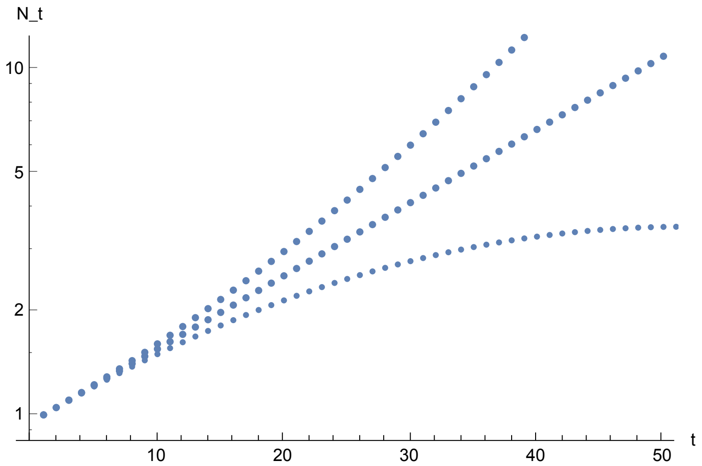


## Let compare the three cases

```
Show[g0, g1, g2]
```




```
Show[gLog0, gLog1, gLog2]
```




# Question

Assume various scenarios about annual growth rate that changes with time and simulate it.

For example, annual growth rate of the world population was 1.07% in 2015. If it linearly decreases by e.g., 0.02% every year, how does the world pouplation grow in future? What is the maximal world population in this case?

Annual growth rate of Japanese population was - 0.17 % in 2015. If it linearly increases by e.g., 0.02 % every year, when does Japanese population start to increase? What is the minimal Japanese population in this case?

What else would be interesting? Simulate them.

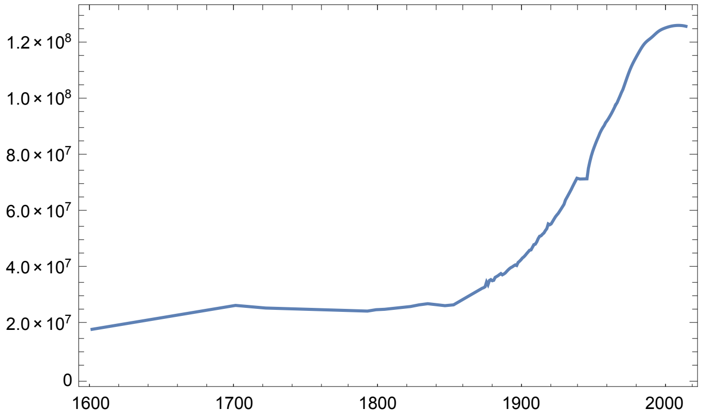
## Example: How the world population has been changing in the past decades

```
data = CountryData["Japan", {"Population", All}]
```

```
TimeSeries[ Time: 01 Jan 1600 to 01 Jan 2014  Data points 149 ]
```

We have data from 1970 to 2013.

```
DateListPlot[data]
```

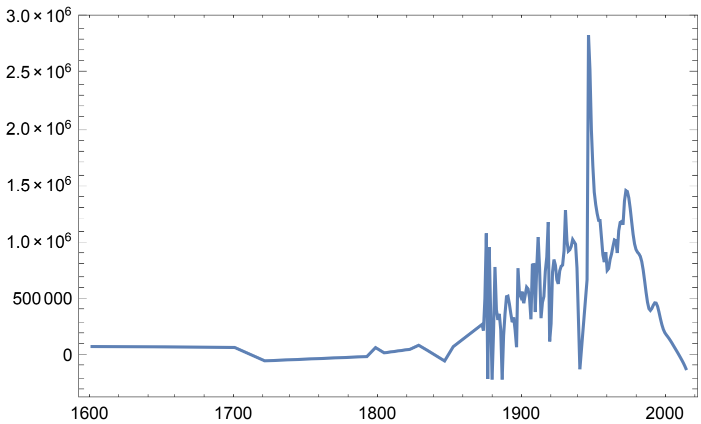



Calculate "difference" between years, i.e., how many people had increased in a year.

```
dataDiff = Differences[data]
```

```
TimeSeries[ Time: 31 Dec 1600 to 22 Sep 2013  Data points 149 ]
```

```
DateListPlot[dataDiff]
```

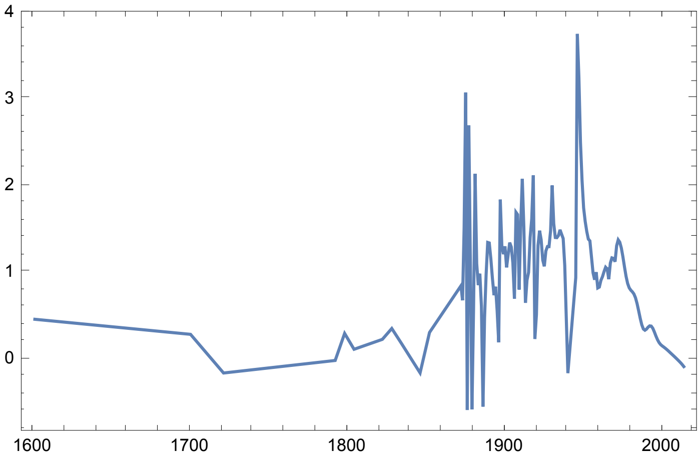



Calculate annual growth rate in percentage, i.e., relative growth.

```
annGrowthRate = dataDiff / data 100
```

```
TimeSeries[ Time: 31 Dec 1600 to 22 Sep 2013  Data points 149 ]
```

```
DateListPlot[annGrowthRate]
```




```
annGrowthRate["2000"] // N
```

0.150314

```
annGrowthRate["2013"] // N
```

-0.071289

```
annGrowthRate["1971"]
```

2.01111

```
(annGrowthRate["2000"] - annGrowthRate["2013"]) / 13 // N
```

0.0170464

Annaul growth rate of the world is constantly decreasing since around 1990. If this trend continues, how large will be the world population in future?

## Obsolete