Stochastic immigration-emigration model

Integer-valued population size changes stochastically with time. With the rate ALPHA, pop size increases by one. With the rate BETA, it decreases by one. We have implemented this stochastic process in continuous time in a C program and output the results to data files. Here we visualize the results.

```
SetDirectory[
 "/Users/takasu/Desktop/test/DerivedData/test/Build/Products/Debug/"]
```

```
/Users/takasu/Desktop/test/DerivedData/test/Build/Products/Debug
```

Read the data file with variable time intervals.

```
data = ReadList["data", Real, RecordLists → True];
Length[data]
data2 = Map[Partition[#, 2] &, data]
```

```
500
```

```
{ ⋯ 1 ⋯ }
```

大きい出力 | 表示を少なく | もっと表示 | すべて表示 | 大きさ制限の設定...

Read the data file with an equal time interval DT = 1.0

```
dataEq = ReadList["data_eq", Real, RecordLists → True];
Length[dataEq]
dataEq2 = Map[Partition[#, 2] &, dataEq]
```

```
500
```

```
{{{0., 10.}, {1., 10.}, {2., 9.}, {3., 10.}, {4., 10.}, {5., 9.},
  {6., 8.}, {7., 8.}, {8., 8.}, {9., 8.}, ⋯ 81 ⋯, {91., 12.},
  {92., 14.}, {93., 15.}, {94., 15.}, {95., 16.}, {96., 16.}, {97., 16.},
  {98., 15.}, {99., 15.}, {100., 14.}}, ⋯ 498 ⋯, { ⋯ 1 ⋯ }}
```

大きい出力 | 表示を少なく | もっと表示 | すべて表示 | 大きさ制限の設定...

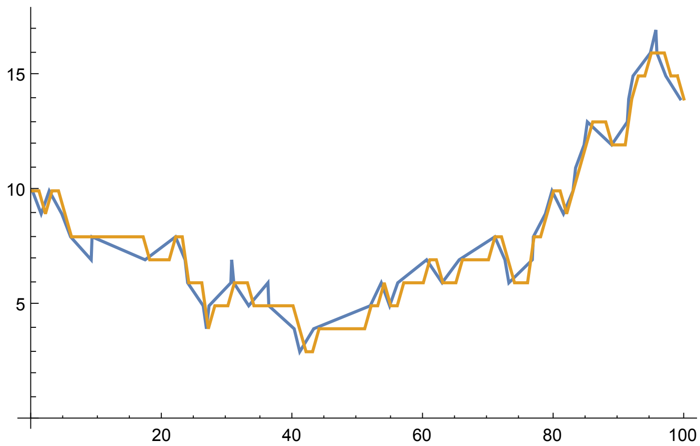
```
data2[[1]]
```

```
{{0., 10.}, {1.34386, 9.}, {2.5757, 10.}, {4.50961, 9.}, {5.85973, 8.},
 {9.02449, 7.}, {9.15487, 8.}, {17.2986, 7.}, {21.9632, 8.},
 {23.4353, 7.}, {23.8288, 6.}, {26.2126, 5.}, {26.7127, 4.}, {27.111, 5.},
 {30.4595, 6.}, {30.5864, 7.}, {30.9113, 6.}, {33.2002, 5.}, {36.1525, 6.},
 {36.2854, 5.}, {40.1765, 4.}, {41.0354, 3.}, {43.1717, 4.}, {51.859, 5.},
 {53.5465, 6.}, {54.8312, 5.}, {56.0772, 6.}, {60.5085, 7.}, {62.8888, 6.},
 {65.5057, 7.}, {70.9792, 8.}, {72.5251, 7.}, {73.1326, 6.}, {76.731, 7.},
 {76.89, 8.}, {78.738, 9.}, {79.7538, 10.}, {81.5093, 9.}, {82.979, 10.},
 {83.3729, 11.}, {84.6795, 12.}, {85.1834, 13.}, {88.9086, 12.},
 {91.3131, 13.}, {91.5243, 14.}, {92.1884, 15.}, {94.8491, 16.},
 {95.6976, 17.}, {95.8522, 16.}, {97.219, 15.}, {99.4381, 14.}}
```

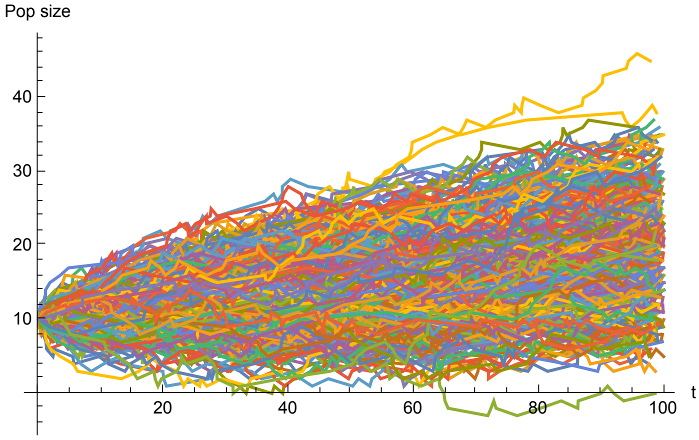
```
dataEq2[[1]]
```

```
{{0., 10.}, {1., 10.}, {2., 9.}, {3., 10.}, {4., 10.}, {5., 9.}, {6., 8.},
 {7., 8.}, {8., 8.}, {9., 8.}, {10., 8.}, {11., 8.}, {12., 8.}, {13., 8.},
 {14., 8.}, {15., 8.}, {16., 8.}, {17., 8.}, {18., 7.}, {19., 7.}, {20., 7.},
 {21., 7.}, {22., 8.}, {23., 8.}, {24., 6.}, {25., 6.}, {26., 6.}, {27., 4.},
 {28., 5.}, {29., 5.}, {30., 5.}, {31., 6.}, {32., 6.}, {33., 6.}, {34., 5.},
 {35., 5.}, {36., 5.}, {37., 5.}, {38., 5.}, {39., 5.}, {40., 5.}, {41., 4.},
 {42., 3.}, {43., 3.}, {44., 4.}, {45., 4.}, {46., 4.}, {47., 4.}, {48., 4.},
 {49., 4.}, {50., 4.}, {51., 4.}, {52., 5.}, {53., 5.}, {54., 6.}, {55., 5.},
 {56., 5.}, {57., 6.}, {58., 6.}, {59., 6.}, {60., 6.}, {61., 7.}, {62., 7.},
 {63., 6.}, {64., 6.}, {65., 6.}, {66., 7.}, {67., 7.}, {68., 7.}, {69., 7.},
 {70., 7.}, {71., 8.}, {72., 8.}, {73., 7.}, {74., 6.}, {75., 6.}, {76., 6.},
 {77., 8.}, {78., 8.}, {79., 9.}, {80., 10.}, {81., 10.}, {82., 9.},
 {83., 10.}, {84., 11.}, {85., 12.}, {86., 13.}, {87., 13.}, {88., 13.},
 {89., 12.}, {90., 12.}, {91., 12.}, {92., 14.}, {93., 15.}, {94., 15.},
 {95., 16.}, {96., 16.}, {97., 16.}, {98., 15.}, {99., 15.}, {100., 14.}}
```

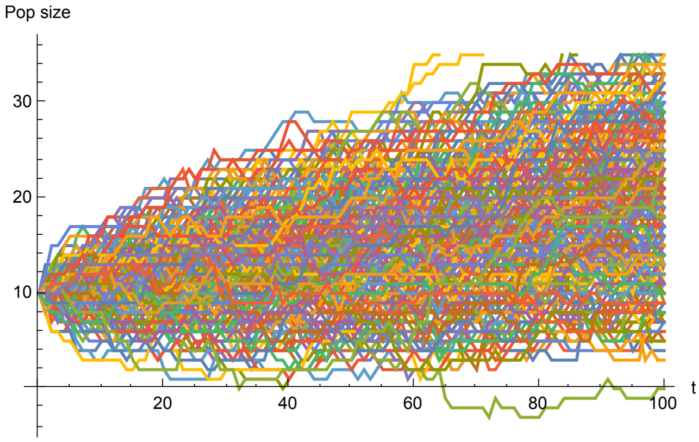
```
ListPlot[{data2[[1]], dataEq2[[1]]}, Joined → True]
```



```
ListPlot[data2, Joined → True, AxesLabel → {"t", "Pop size"}]
```




```
gSim = ListPlot[dataEq2, Joined → True, AxesLabel → {"t", "Pop size"}]
```



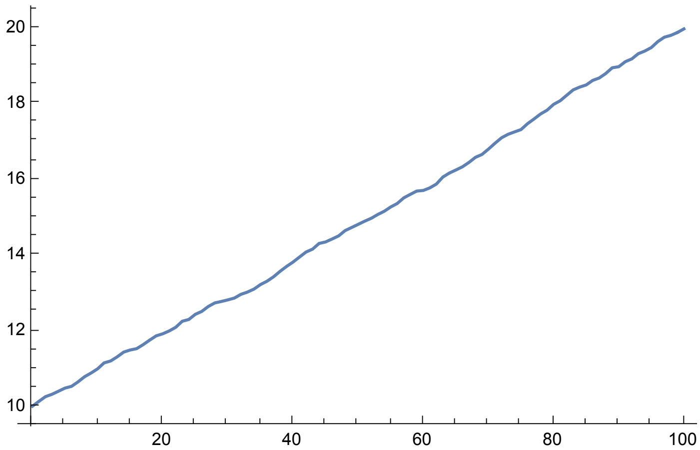
It is easy to use the data with an equal time interval to calculate average pop size at each time.

```
popSeqList = {};
Do[
 tmp = Transpose[dataEq2[[i]]];
AppendTo[popSeqList, tmp[[2]]], {i, 1, Length[dataEq2]}
]
```

```
timeSeq = Transpose[dataEq2[[1]]][[1]];
popSeqAverage = Map[Mean, Transpose[popSeqList]];
seqAverage = {timeSeq, popSeqAverage} // Transpose
```

```
{{0., 10.}, {1., 10.128}, {2., 10.252}, {3., 10.314}, {4., 10.396}, {5., 10.478},
 {6., 10.524}, {7., 10.642}, {8., 10.778}, {9., 10.876}, {10., 10.986},
 {11., 11.144}, {12., 11.194}, {13., 11.302}, {14., 11.428}, {15., 11.484},
 {16., 11.52}, {17., 11.626}, {18., 11.746}, {19., 11.858}, {20., 11.912},
 {21., 11.988}, {22., 12.084}, {23., 12.244}, {24., 12.29}, {25., 12.426},
 {26., 12.504}, {27., 12.634}, {28., 12.728}, {29., 12.766}, {30., 12.808},
 {31., 12.856}, {32., 12.954}, {33., 13.014}, {34., 13.092}, {35., 13.212},
 {36., 13.3}, {37., 13.416}, {38., 13.56}, {39., 13.69}, {40., 13.806},
 {41., 13.94}, {42., 14.072}, {43., 14.148}, {44., 14.298}, {45., 14.338},
 {46., 14.416}, {47., 14.498}, {48., 14.638}, {49., 14.72}, {50., 14.802},
 {51., 14.886}, {52., 14.962}, {53., 15.062}, {54., 15.148}, {55., 15.262},
 {56., 15.354}, {57., 15.502}, {58., 15.594}, {59., 15.682}, {60., 15.698},
 {61., 15.766}, {62., 15.866}, {63., 16.052}, {64., 16.158}, {65., 16.238},
 {66., 16.322}, {67., 16.438}, {68., 16.572}, {69., 16.648}, {70., 16.79},
 {71., 16.944}, {72., 17.088}, {73., 17.18}, {74., 17.244}, {75., 17.306},
 {76., 17.462}, {77., 17.586}, {78., 17.718}, {79., 17.818}, {80., 17.972},
 {81., 18.066}, {82., 18.21}, {83., 18.354}, {84., 18.428}, {85., 18.484},
 {86., 18.604}, {87., 18.668}, {88., 18.786}, {89., 18.938}, {90., 18.966},
 {91., 19.098}, {92., 19.172}, {93., 19.31}, {94., 19.38}, {95., 19.472},
 {96., 19.63}, {97., 19.746}, {98., 19.794}, {99., 19.87}, {100., 19.964}}
```

```
gAve = ListPlot[seqAverage, Joined → True]
```

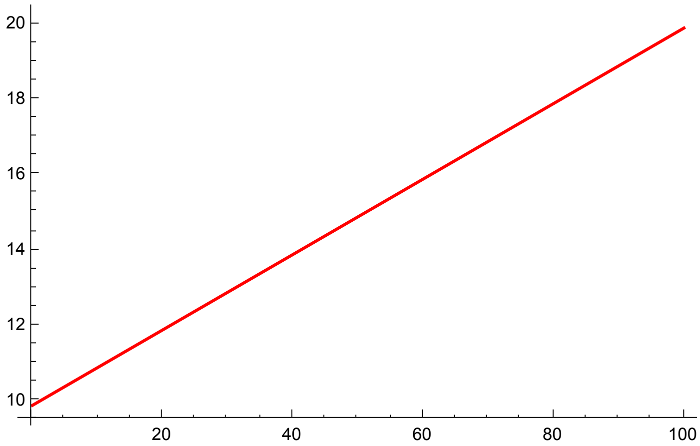



```
fitted = Fit[seqAverage, {1, t}, t]
```

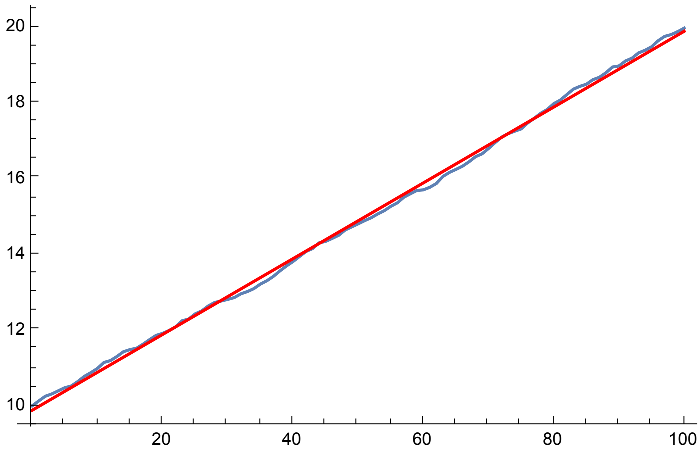
```
9.88324 + 0.0999763 t
```

How are the coefficients related to the parameter used in the process?

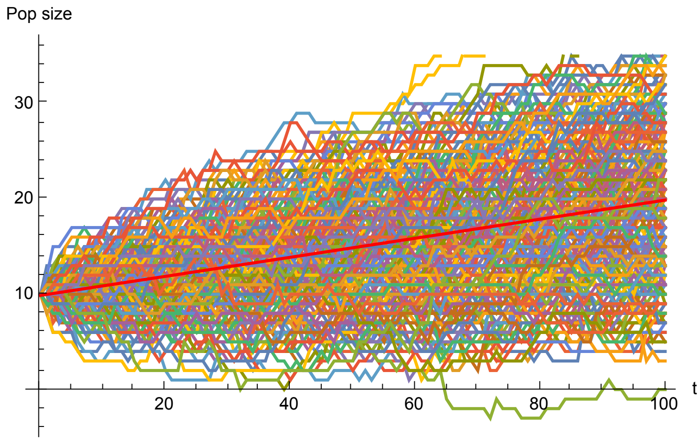
```
gFitted = Plot[fitted, {t, 0, 100}, PlotStyle → RGBColor[1, 0, 0]]
```




```
Show[gAve, gFitted]
```



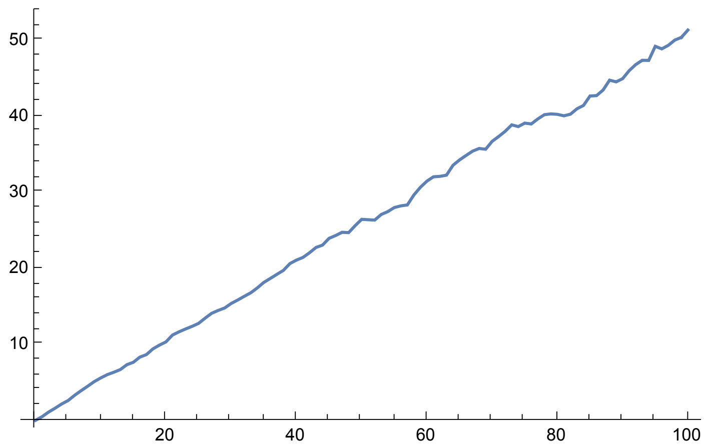

```
Show[gSim, gFitted]
```



```
timeSeq = Transpose[dataEq2[[1]]][[1]];
popSeqVariance = Map[Variance, Transpose[popSeqList]];
seqVariance = {timeSeq, popSeqVariance} // Transpose
```

```
{{0., 0.}, {1., 0.476569}, {2., 1.08667}, {3., 1.59459}, {4., 2.14748},
 {5., 2.60673}, {6., 3.29602}, {7., 3.90565}, {8., 4.48569}, {9., 5.09081},
 {10., 5.57696}, {11., 6.01129}, {12., 6.32501}, {13., 6.67615}, {14., 7.31545},
 {15., 7.63702}, {16., 8.31824}, {17., 8.64341}, {18., 9.40028}, {19., 9.89362},
 {20., 10.3169}, {21., 11.2183}, {22., 11.6402}, {23., 12.0165}, {24., 12.3586},
 {25., 12.754}, {26., 13.4369}, {27., 14.0762}, {28., 14.4509}, {29., 14.7768},
 {30., 15.3699}, {31., 15.8229}, {32., 16.3085}, {33., 16.7754}, {34., 17.4144},
 {35., 18.1393}, {36., 18.6633}, {37., 19.1973}, {38., 19.7218}, {39., 20.6151},
 {40., 21.0745}, {41., 21.4232}, {42., 22.0389}, {43., 22.7316}, {44., 23.0473},
 {45., 23.9356}, {46., 24.2995}, {47., 24.7315}, {48., 24.6843}, {49., 25.6008},
 {50., 26.4357}, {51., 26.3818}, {52., 26.3372}, {53., 27.0723}, {54., 27.449},
 {55., 27.9813}, {56., 28.2051}, {57., 28.3226}, {58., 29.6364}, {59., 30.6462},
 {60., 31.4537}, {61., 32.0273}, {62., 32.0842}, {63., 32.2418}, {64., 33.5441},
 {65., 34.2418}, {66., 34.832}, {67., 35.3969}, {68., 35.7523}, {69., 35.6514},
 {70., 36.6913}, {71., 37.3235}, {72., 38.0123}, {73., 38.8573}, {74., 38.6297},
 {75., 39.0865}, {76., 38.9745}, {77., 39.6379}, {78., 40.1909}, {79., 40.2975},
 {80., 40.2397}, {81., 40.0457}, {82., 40.2584}, {83., 40.9706},
 {84., 41.4076}, {85., 42.6551}, {86., 42.7006}, {87., 43.4326},
 {88., 44.7337}, {89., 44.5112}, {90., 44.9307}, {91., 45.9804}, {92., 46.776},
 {93., 47.3446}, {94., 47.3303}, {95., 49.1836}, {96., 48.8508},
 {97., 49.3121}, {98., 50.0036}, {99., 50.3578}, {100., 51.3173}}
```

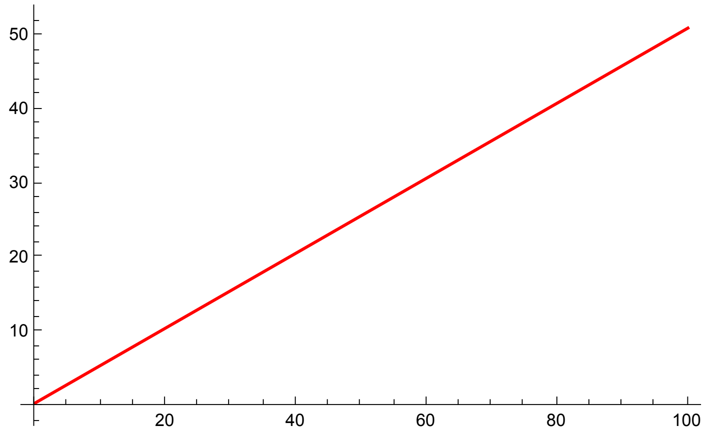
```
gVar = ListPlot[seqVariance, Joined → True]
```



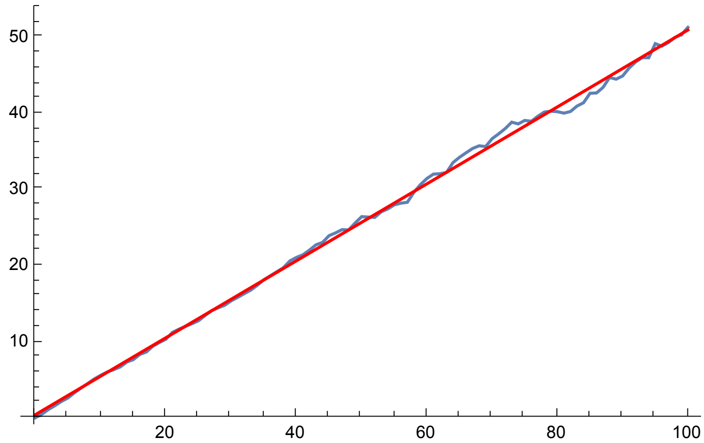

```
fitted = Fit[seqVariance, {1, t}, t]
```

```
0.404562 + 0.505743 t
```

```
gVarFitted = Plot[fitted, {t, 0, 100}, PlotStyle → RGBColor[1, 0, 0]]
```




```
Show[gVar, gVarFitted]
```



How are the coefficients related to the parameter used in the process?

We will learn how the simunlation results can be analytically interpreted.